



fichas de Trabajo

MATEMÁTICA

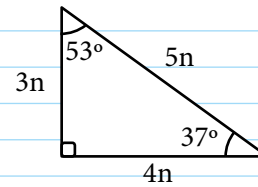
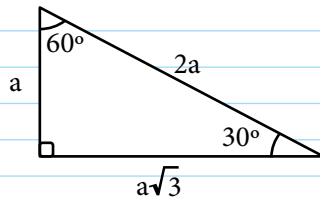
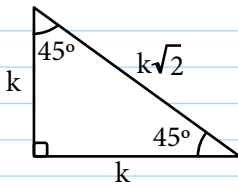
3^{ro}

SECUNDARIA

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS NOTABLES

Los triángulos rectángulos de ángulos notables o simplemente triángulos rectángulos notables, son aquellos en los cuales conociendo las medidas de sus ángulos agudos se puede saber en qué proporción se encuentran sus lados.

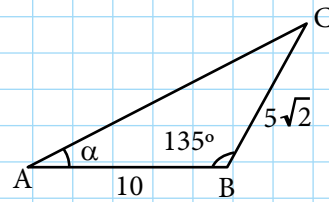
Destacan:



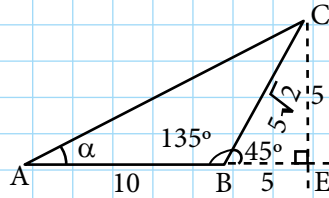
A partir de estos triángulos, se calculan las razones trigonométricas de sus correspondientes ángulos:

	$45^\circ < > \frac{\pi}{4}$	$30^\circ < > \frac{\pi}{6}$	$60^\circ < > \frac{\pi}{3}$	37°	53°
Sen	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{3}{5}$	$\frac{4}{5}$
Cos	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{3}{5}$
Tan	1	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{3}{4}$	$\frac{4}{3}$
Cot	1	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$\frac{4}{3}$	$\frac{3}{4}$
Sec	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	2	$\frac{5}{4}$	$\frac{5}{3}$
Csc	$\sqrt{2}$	2	$\frac{2}{\sqrt{3}}$	$\frac{5}{3}$	$\frac{5}{4}$

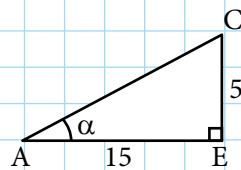
Calcula $\tan \alpha$



Resolución:

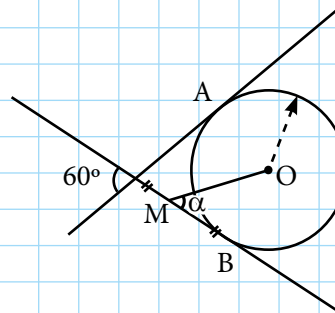


Del triángulo ACE

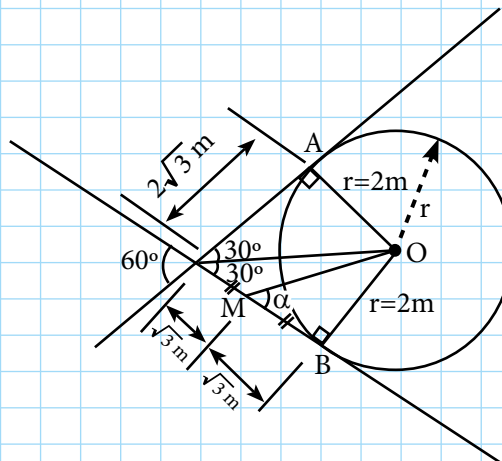


$$\tan \alpha = \frac{5}{15} = \frac{1}{3}$$

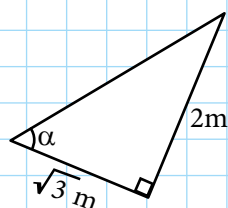
Calcula $\cot \alpha$



Resolución:

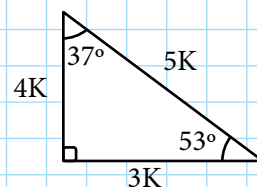
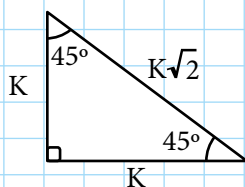
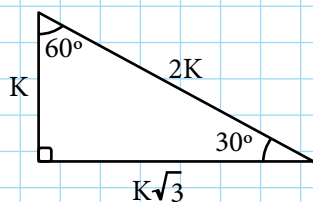


Sea $r = 2m$
Del triángulo MBO.



$$\cot \alpha = \frac{\sqrt{3}m}{2m} = \frac{\sqrt{3}}{2}$$

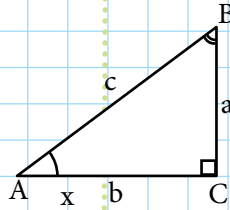
ESQUEMA FORMULARIO



	30°	37°	45°	53°	60°
Sen	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{\sqrt{2}}{2}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{\sqrt{2}}{2}$	$\frac{3}{5}$	$\frac{1}{2}$
Tan	$\frac{\sqrt{3}}{3}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$
Cot	$\sqrt{3}$	$\frac{4}{3}$	1	$\frac{3}{4}$	$\frac{\sqrt{3}}{3}$
Sec	$\frac{2\sqrt{3}}{3}$	$\frac{5}{4}$	$\sqrt{2}$	$\frac{5}{3}$	2
Csc	2	$\frac{5}{3}$	$\sqrt{2}$	$\frac{5}{4}$	$\frac{2\sqrt{3}}{3}$

Propiedades de las razones trigonométricas

Tomando un triángulo ABC recto en C como referencia:



I. Razones recíprocas (inversas)

Son aquellas parejas de razones trigonométricas cuyos valores son inversos, por ejemplo:

$$\text{Sen} \alpha = \frac{a}{c} \quad \wedge \quad \text{Csc} \alpha = \frac{c}{a}$$

$$\Rightarrow \text{Sen} \alpha \cdot \text{Csc} \alpha = \frac{a}{c} \cdot \frac{c}{a} = 1$$

En conclusión:

$$\text{Sen} \alpha \cdot \text{Csc} \alpha = 1$$

$$\text{Cos} \alpha \cdot \text{Sec} \alpha = 1$$

$$\text{Tan} \alpha \cdot \text{Cot} \alpha = 1$$

Ángulos iguales

II. Razones complementarias (co - razones)

Se caracterizan por tener igual valor numérico solo si sus ángulos suman 90° , por ejemplo:

$$\text{Sen} A = \frac{a}{c} \quad \wedge \quad \text{Cos} B = \frac{a}{c}$$

$$\Rightarrow \text{Sen} A = \text{Cos} B$$

En conclusión:

$$\text{Sen} A = \text{Cos} B$$

$$\text{Tan} A = \text{Cot} B$$

$$\text{Sec} A = \text{Csc} B$$

$$A + B = 90^\circ$$

También se puede afirmar:

$$\text{R.T.}(\theta) = \text{Co - R.T.}(90^\circ - \theta)$$

1. Halla $\text{Sen}(x + 12^\circ)$, si: $\text{Tan} x \cdot \text{Tan} 72^\circ = 1$

Resolución:

$$\text{Tan} x \cdot \text{Tan} 72^\circ = 1$$

$$\text{Tan} x \cdot \text{Cot} 18^\circ = 1 \Rightarrow x = 18^\circ$$

$$\text{Piden: } \text{Sen}(x + 12^\circ) = \text{Sen} 30^\circ = \frac{1}{2}$$

2. Calcula: $E = (2\text{Sen} 70^\circ + \text{Cos} 20^\circ)(\text{Sec} 20^\circ + \text{Csc} 70^\circ)$

Resolución:

$$E = (2\text{Sen} 70^\circ + \text{Cos} 20^\circ)(\text{Sec} 20^\circ + \text{Csc} 70^\circ)$$

$$E = (2\text{Sen} 70^\circ + \text{Sen} 70^\circ)(\text{Csc} 70^\circ + \text{Csc} 70^\circ)$$

$$E = (3\text{Sen} 70^\circ)(2\text{Csc} 70^\circ)$$

$$E = 6\text{Sen} 70^\circ \text{Csc} 70^\circ$$

$$E = 6(1)$$

$$E = 6$$

● Trabajando en Clase

1. Calcula:

$$M = \tan 45^\circ + 2\cos 60^\circ - 8\csc 53^\circ$$

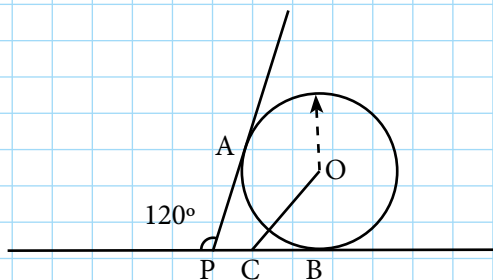
2. Calcula:

$$E = \frac{(\sec 45^\circ)^{\sec 60^\circ} + (\cot 30^\circ)^{\sec 60^\circ}}{4\tan 37^\circ}$$

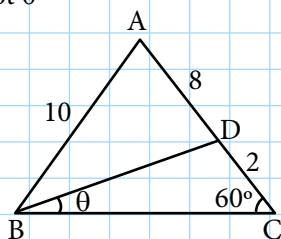
3. Halla el valor de « α » sabiendo que es un ángulo agudo.

$$\tan(\alpha + 5^\circ) = \sqrt{2\sec 30^\circ + \sec 245^\circ}$$

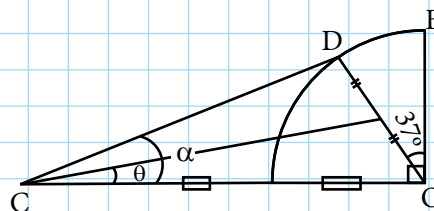
4. Calcula $\tan \Psi$ ($BC = 2PC$)



5. Calcula $\cot \theta$



6. Calcula: $E = \cot \theta - \cot \alpha$



7. Calcula « x » si:

$$\cos(3x - 12^\circ) \cdot \sec(x + 36^\circ) = 1$$

8. Calcula « y » si:

$$\sin(y + 10^\circ) = \cos(y + 20^\circ)$$

9. Calcula $\cos 3x$, si:

$$\tan(5x) \cdot \cot(x + 40^\circ) = 1$$

10. Calcula:

$$(4\sin 26^\circ + 3\cos 64^\circ)(\csc 26^\circ + 2\sec 64^\circ)$$

11. Si se cumple:

$$\tan\left[\cot\left(\frac{\theta}{2}\right)\right] \cdot \cot[\tan(2\theta)] = 1$$

$$\text{Calcula: } K = \frac{\tan(\theta + 1^\circ)}{2} - \tan(\theta + 9^\circ)$$

12. Si: $\sin 2x \cdot \cos(37^\circ + x) = \sin(53^\circ - x) \cdot \cos 3x$

$$\text{Calcula: } N = \tan^2(3x + 6^\circ) + \cot^2(2x + 9^\circ)$$

Tarea domiciliaria N° 4

1. Calcula: $J = 5\text{Sen}53^\circ - \text{Tan}45^\circ + 2\text{Cos}60$

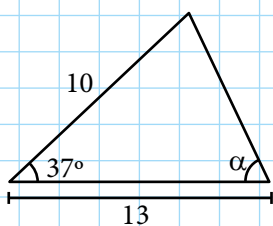
Rpta :

2. Calcula:

$$\frac{(\text{Csc}45^\circ)^{\text{Csc}30^\circ} + (\text{Tan}60^\circ)^{\text{Csc}30^\circ}}{5\text{Cot}53^\circ}$$

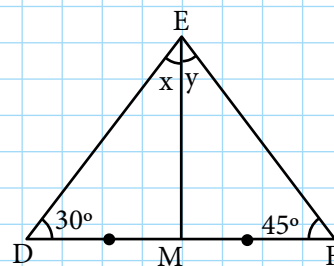
Rpta :

3. Calcula $\text{Sen}\alpha$



Rpta :

4. Calcula $\text{Cot}\psi$



Rpta :

5. Calcula «y» si: $\sec(y + 42^\circ) = \csc(2y + 30^\circ)$

Rpta :

6. Calcula «2x» si: $\cos(3x + 12^\circ) \sec(2x + 30^\circ) = 1$

Rpta :

7. Calcula:

$$C = \frac{5\cot 72^\circ}{\tan 18^\circ} + \frac{\sec 35^\circ}{\csc 55^\circ} - \frac{8\sin 20^\circ}{\cos 70^\circ}$$

Rpta :

8. Si $\cos(80^\circ - 3x) \cdot \sec(5x) = 1$

Calcula: $M = \sec^2\left(\frac{9x}{2}\right) + \sqrt{3} \tan 3x + 2$

Rpta :